* How to represent data by basis vectors?

$\bar{x} \in R^d$

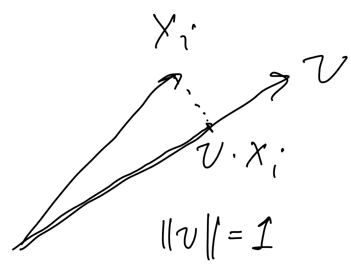
$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} = x_1 \cdot \vec{d}_1 + x_2 \cdot \vec{d}_2 + \cdots + x_d \cdot \vec{d}_d$$

$$= x_1 \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + x_2 \cdot \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} + \cdots + x_d \cdot \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}$$

basis vectors

⇒ How to get the coordinate of a new vector $\bar{y}$ from those basis?

$$\underbrace{\bar{y} \cdot \vec{d}_1}_{\text{inner product}} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_d \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = y_1$$



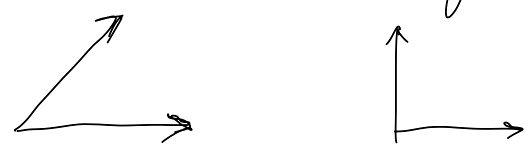
⇒ What if we switch basis?

$$\bar{y} \cdot \bar{m}_1 = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_d \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_d \end{bmatrix} = y_1^*$$

$\bar{m}_1$ is the new basis vector.

⇒ Which vectors can be basis vectors?

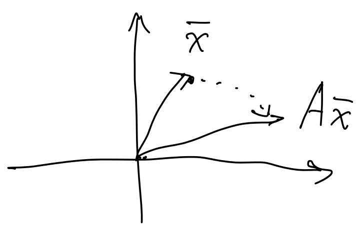
They have to be linearly independent,
doesn't have to be orthogonal.



both are fine.

* Linear transform:

$$A\bar{x}, \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}$$



We will talk more about this in 3D vision.